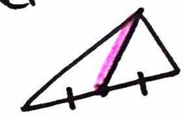
Name: Date: Period: 6

Medians and Altitudes of Triangles

Lesson Objective IWBAT find and use centroids and orthocenters of Δs.



Median: A median of a triangle is a segment from a vertex to the midpoint of the opposite side

The three medians of the triangle are *concurrent*. This point of concurrency is called the centroid.

The ratio of **larger : smaller** segments within the median is 2 : 1. DEF

What would the ratio of the larger segment of the median to the ENTIRE median be?

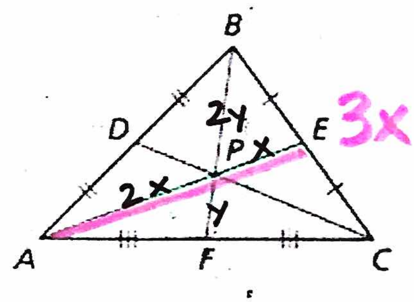
2 : 3

Centroid Theorem

The centroid of a triangle is $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.

The medians of ΔABC meet at point P, and…

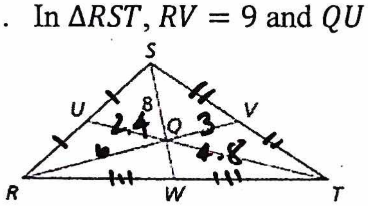
$$\frac{AP}{AE} = \frac{2}{3} \quad \dots \quad \frac{AP}{PE} = \frac{2}{1} \quad \dots \quad \frac{FP}{FB} = \frac{1}{3}$$



The centroid is also called the "center of gravity" or "balance point"

Example:

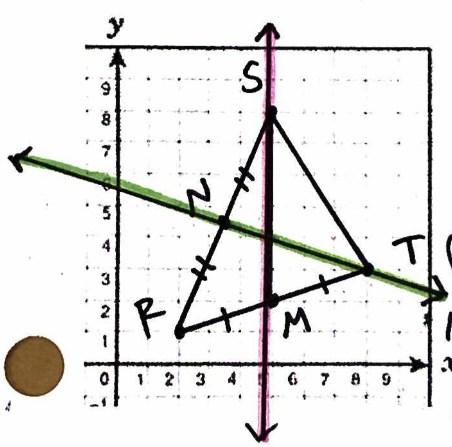
1. In ΔRST, $RV = 9$ and $QU = 2.4$. Find RQ and TU.



RQ = 6

TU = 7.2

2. Find the coordinates of the centroid of Δ RST where R(2, 1), S(5, 8), T(8, 3).



$x = 5$

$y = -\frac{1}{3}x + \frac{17}{3}$

↳ USE DEFINITION: find eqns of 2 medians. then solve system of eqns.

from S:

① mdpt of $\overline{RT}$

$M\left(\frac{2+8}{2}, \frac{1+3}{2}\right) = (5, 2)$

② vertical line!

$x = 5$

from T:

① mdpt of $\overline{RS}$

$\left(\frac{2+5}{2}, \frac{1+8}{2}\right) = (3.5, 4.5) N$

② slope of $\overline{TN}$

$\frac{3-4.5}{8-3.5} = \frac{-1.5}{4.5} = \frac{-15}{45} = -\frac{1}{3}$

③ eqn. of $\overline{TN}$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{1}{3}(x - 8)$$

$$y - 3 = -\frac{1}{3}x + \frac{8}{3}$$

$$+3 \qquad\qquad +3$$

$$y = -\frac{1}{3}x + \frac{8}{3} + \frac{9}{3}$$

$$y = -\frac{1}{3}x + \frac{17}{3}$$

Solve system:

$$y = -\frac{1}{3}(5) + \frac{17}{3}$$

$$y = -\frac{5}{3} + \frac{17}{3}$$

$$y = \frac{12}{3}$$

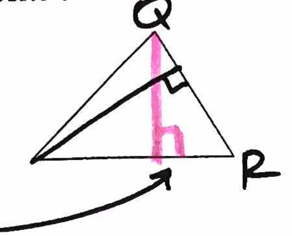
$$y = 4$$

$$\boxed{(5, 4)}$$

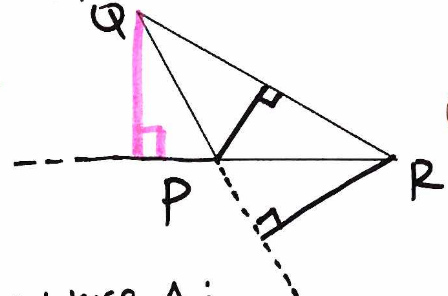
(height)

Altitude: An altitude of a triangle is the $\perp$ segment from a vertex to the opposite side OR the line that contains the opposite side.

*not necessarily @ mdpt



Altitude from Q to $\overleftrightarrow{PR}$

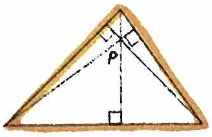


The lines containing the altitudes of a triangle are concurrent.

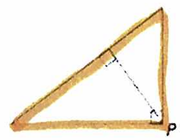
This point of concurrency is the orthocenter of the triangle.

*obtuse Δ: altitudes from acute ∠s are outside Δ

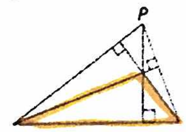
The location of the orthocenter (P, in the diagram below) depends on the type of triangle.



acute Δ: inside



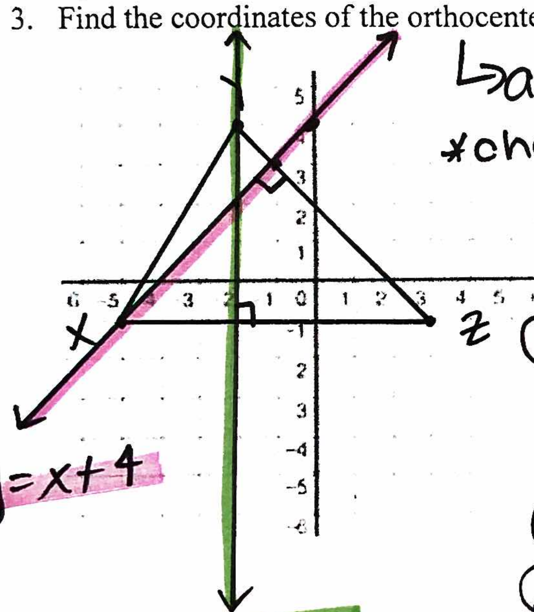
right Δ: on rt. ∠ vertex



obtuse Δ: outside

Example:

3. Find the coordinates of the orthocenter of ΔXYZ with vertices X(-5, -1), Y(-2, 4) and Z(3, -1)



↳ altitudes: thru vertex, ⊥ to opp. side

*choose any vert./horiz. sides to work w/ first (if you have them)

alt. from X:

① slope of $\overline{YZ}$

$\frac{-1-4}{3-(-2)} = \frac{-5}{5} = -1$

② ⊥ slope = 1

③ eqn. of alt.

$y-(-1) = 1(x-(-5))$

$y+1 = x+5 \rightarrow y = x+4$

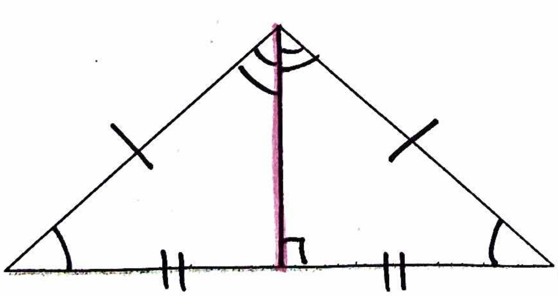
solve system

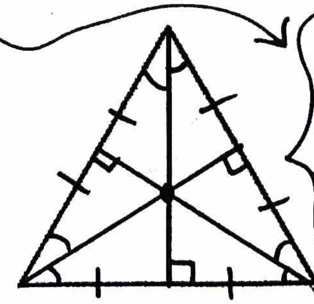
$y = -2+4$

$y = 2$

$\boxed{(-2, 2)}$

In an **isosceles** Δ, the perpendicular bisector, angle bisector, median, and altitude **from the vertex angle to the base** are all the **same segment.** In an **equilateral** Δ, this is true for **any vertex**.





centroid
orthocenter
circumcenter
incenter

are all the same point!

See pg 323 for summary chart for circumcenter, incenter, centroid and orthocenter